# Fundamentals of Traffic Flow

Masoud Ghodrat Abadi, PhD
Rohollah (Roham) Moghadam, PhD

WAVE
WORKFORCE FOR AUTONOMOUS VEHICLE ENGINEERING

SACRAMENTO STATE

MTI MINETA TRANSPORTATION INSTITUTE

# Learning Objectives

1. **Define** key traffic stream parameters, including speed, flow, and density
2. **Differentiate** between time-mean speed (TMS) and space-mean speed (SMS), and **calculate** TMS and SMS using given data sets and formulas
3. **Calculate** traffic flow and density based on observed headways and spacings
4. **Apply** the fundamental relationship among speed, flow, and density ($q = u \times k$)
5. **Interpret** Greenshield's linear speed–density model and derive flow–density and speed–flow relationships
6. **Evaluate** how theoretical models compare to real-world traffic behavior and assess their limitations

# Introduction

- Streams of traffic are comprised of individual vehicles, piloted by individual drivers, interacting with each other and the roadway environment (geometry, traffic control devices, etc.).
- Because each driver behaves in a unique way (i.e., has its own brain), it is not possible to describe traffic flow as theoretically concisely as other purely physical phenomena.
- But we still need quantitative techniques to assess operational measures of highway performance, so we can analyze and evaluate existing transportation facilities, and design improvements for those facilities.

# Terminology

- The three main variables (parameters) that form the underpinning of traffic analysis are:

**Speed (u)**

**Flow (q)**

**Density (k)**

# Speed (u)

- Speed (*u)* - the distance traveled per amount of time *t* (mph)
- Average traffic speed is defined in two ways:

**Time (spot) mean speed (TMS)** – average speed of all vehicles passing a point on a roadway over a specified time (instantaneous point speed, as taken by a radar gun for example)

$$\bar{u}_t = \frac{\sum_{i=1}^{n} u_i}{n} \qquad \text{(Eq. 5.5)}$$

Where:

$\bar{u}_t$ = time-mean speed in unit distance per unit time,
$u_i$ = spot speed (the speed of the vehicle at the designated point on the highway, as might be obtained using a radar gun) of the $i$th vehicle, and
$n$ = number of measured vehicle spot speeds.

# Speed (u)

**<u>Space mean speed (SMS)</u>** – average speed of all vehicles occupying a given section of roadway over a specified time (essentially, the inverse of travel time for all vehicles over the specified section length)

This second definition of speed is more useful in the context of traffic analysis and is determined on the basis of the time necessary for a vehicle to travel some known length of roadway.

# Speed (u)

$$\bar{u}_s = \frac{l}{\bar{t}} \quad \text{(Eq. 5.6)}$$

Where:

$\bar{u}_s$ = space-mean speed in unit distance per unit time,

$l$ = length of roadway used for travel time measurement of vehicles,

$\bar{t}$ = average vehicle travel time, defined as

$$\bar{t} = \frac{1}{n}\sum_{i=1}^{n} t_i \quad \text{(Eq. 5.7)}$$

Where:

$t_i$ = time necessary for vehicle *i* to travel roadway section of length *l*, and

*n* = number of measured vehicle travel times.

# Flow (q)

- Flow (q*)* – number of vehicles (n) passing a point in time

**Traffic Flow** is defined as

$$q = \frac{n}{t} \qquad \text{(Eq. 5.1)}$$

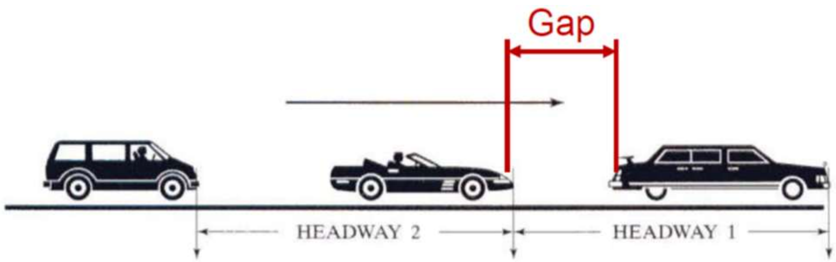
Where:
*q* = traffic flow in vehicles per unit time (veh/hr/ln or vphpl),
*n* = number of vehicles passing some designated roadway point during time *t*, and
*t* = duration of time interval.

# Flow (q)

- **Headway** - The time between the passage of the front bumpers of successive vehicles, at some designated highway point

$$t = \sum_{i=1}^{n} h_i$$

Where:
*t* = duration of time interval,
$h_i$ = time headway of the *i* th vehicle (the time that has transpired between the arrival of vehicle *i* and *i* -1), and
*n* = number of measured vehicle time headways at some designated roadway point.



# Flow (q)

- Substituting Eq. 5.2 into Eq. 5.1 gives

$$q = \frac{n}{\sum_{i=1}^{n} h_i} \quad \text{(Eq. 5.3)} \qquad \text{or} \qquad q = \frac{1}{\bar{h}} \quad \text{(Eq. 5.4)}$$

Where:

$\bar{h}$ is the average time headway, $\sum h_i / n$ in unit time per vehicle.

- Example

q = 2000 veh/h → $\bar{h}$ = 1.8 sec

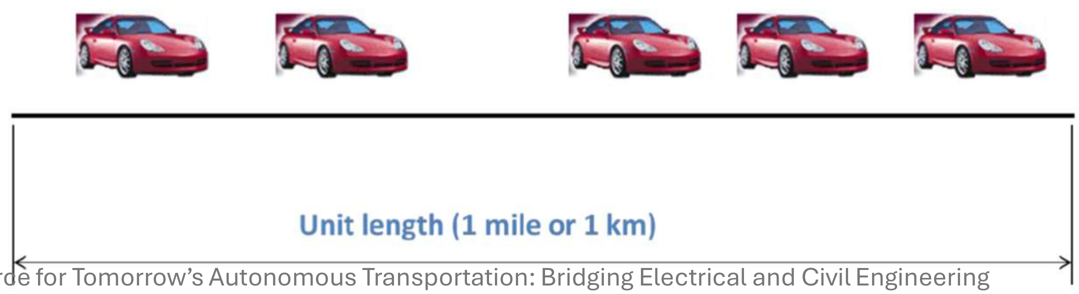
$\bar{h}$ = 3.0 sec → q = 1200 veh/h

# Density (k)

- **Density** – number of vehicles (n) occupying a length of a roadway

$$k = \frac{n}{l} \qquad \text{(Eq. 5.10)}$$

Where:
*k* = traffic density in vehicles per unit distance,
*n* = number of vehicles occupying some length of roadway at some specified time, and
*l* = length of roadway.



# Density (k)

- The density can also be related to the individual spacing between successive vehicles (measured from front bumper to front bumper). The roadway length, *l*, in Eq. 5.10 can be defined as

$$l = \sum_{i=1}^{n} s_i \qquad \text{(Eq. 5.11)}$$

Where:
$s_i$ = spacing of the *i* th vehicle (the distance between vehicle *i* and *i* -1, measured from front bumper to front bumper), and
*n* = number of measured vehicle spacings.

# Density (k)

- Substituting Eq. 5.11 into Eq. 5.10 gives

$$k = \frac{n}{\sum_{i=1}^{n} s_i} \quad \text{(Eq. 5.12)} \qquad \text{or} \qquad k = \frac{1}{\bar{s}} \quad \text{(Eq. 5.13)}$$

Where:

$\bar{S}$ is the average spacing, $\sum s_i / n$ in unit distance per vehicle.

- Example

k = 100 veh/mi → $\bar{s}$ = 52.8 ft

$\bar{s}$ = 40 ft → k = 132 veh/mi

# Variable Relationships

- Based on the definitions presented above, a simple identity provides the basic relationship among traffic flow, speed (space-mean speed), and density (denoting space-mean speed, $\bar{u}_s$, as simply *u* for notational convenience),

$$q = uk \qquad \text{(Eq. 5.14)}$$

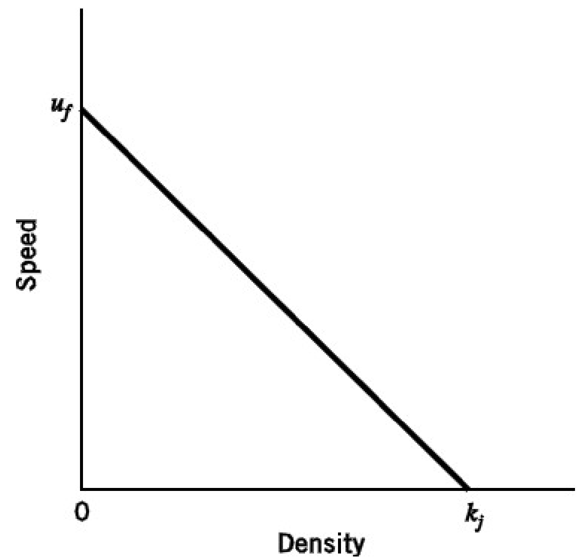
Where:
*q* = flow, typically in units of vehicles per hour (veh/h),
*u* = speed (space mean speed), typically in units of mi/h (km/h), and
*k* = density, typically in units of veh/mi (veh/km).

# Speed – Density Model

Greenshield made the assumption that under uninterrupted flow conditions, speed and density are linearly related (Greenshield's Model)

$$u = u_f \left( 1 - \frac{k}{k_j} \right)$$

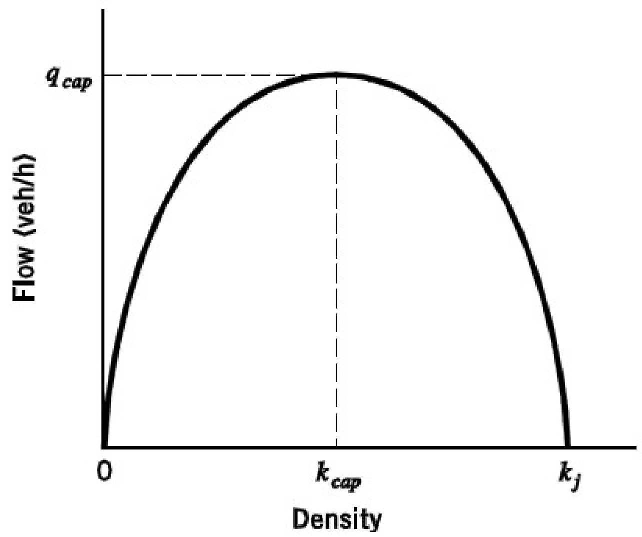
$u$ = space-mean speed in mi/h,
$u_f$ = free-flow speed in mi/h,
$k$ = density in veh/mi, and
$k_j$ = jam density in veh/mi.



# Flow – Density Model

Assuming a linear speed-density relationship (Greenshield's Model) and using q = u · k:

$$q = u_f \left( k - \frac{k^2}{k_j} \right)$$



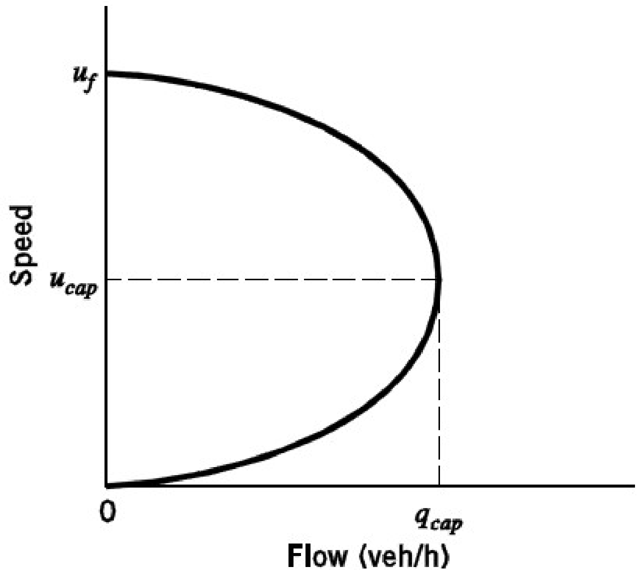

**Density at Capacity: $k_{cap}$ ?**

$$k_{cap} = \frac{k_j}{2}$$

# Speed – Flow Model

By rearranging the original speed-density model using the fundamental traffic flow relationship, $q = u \cdot k$

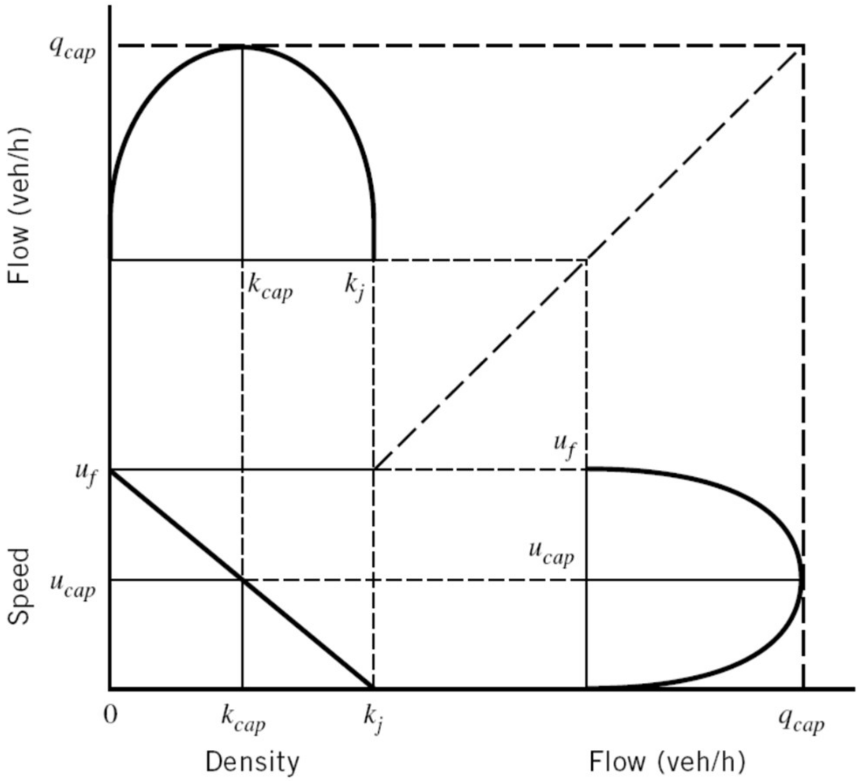
$$q = k_j\left(u - \frac{u^2}{u_f}\right)$$




**Speed at Capacity: $u_{cap}$ ?**

$$u_{cap} = \frac{u_f}{2}$$

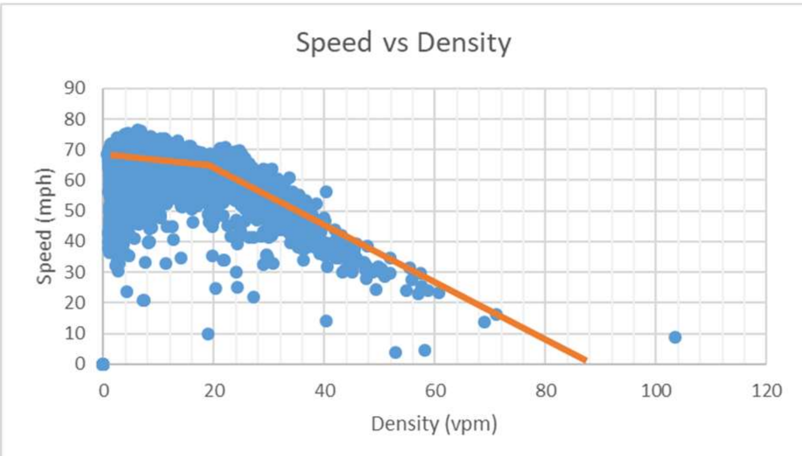
# Theoretical Speed – Flow – Density Relationship

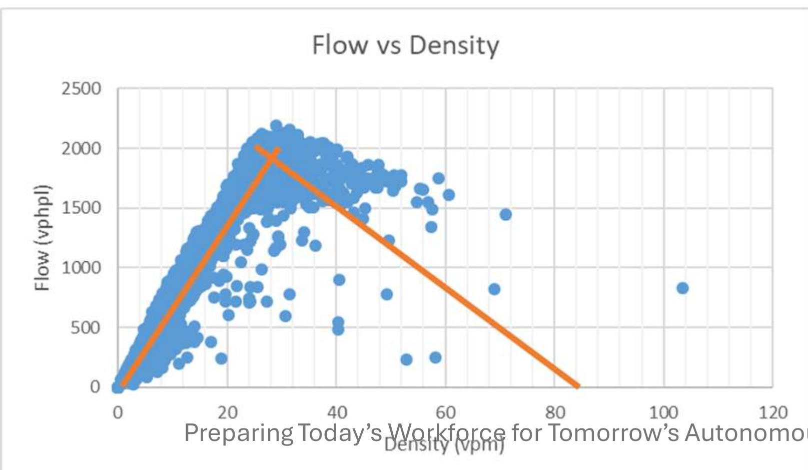


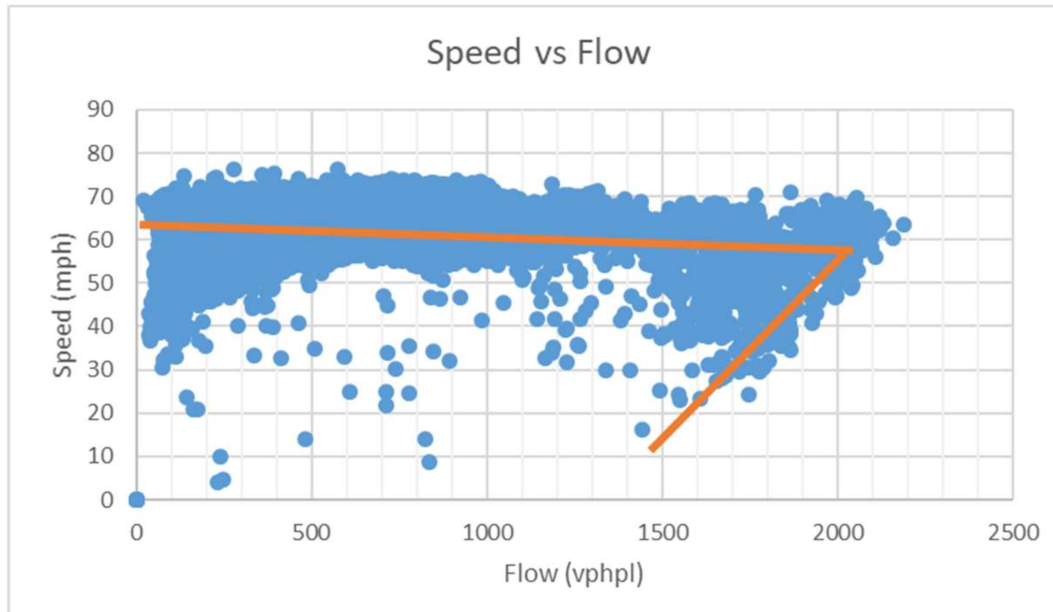

**Flow at Capacity (aka. Capacity): $q_{cap}$ ?**

$$q_{cap} = u_{cap}k_{cap} = \frac{u_f}{2} \times \frac{k_j}{2} = \frac{u_f \times k_j}{4}$$

# Real-World Speed – Flow – Density Relationship



# References


- Mannering, F., and S. Washburn (2020). Principles of Highway Engineering and Traffic Analysis, Seventh Edition, John Wiley & Sons
- Elefteriadou, L. (2024). An Introduction to Traffic Flow Theory, Second Edition, Springer