Control Systems

For Autonomous Transportation

SACRAMENTO STATE

WAVE

WORKFORCE FOR AUTONOMOUS VEHICLE ENGINEERING

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Learning Objectives

- Define fundamental control system concepts and their role in autonomous vehicles
- Understand the role of control systems in autonomous vehicles and why feedback is essential.
- Identify key vehicle states, control inputs, and tracking errors used in AV control.
- Explain how road geometry and civil engineering design influence vehicle control performance.
- Describe how control integrates with perception and planning in the See–Think–Act architecture.

Autonomous Vehicle System Architecture:

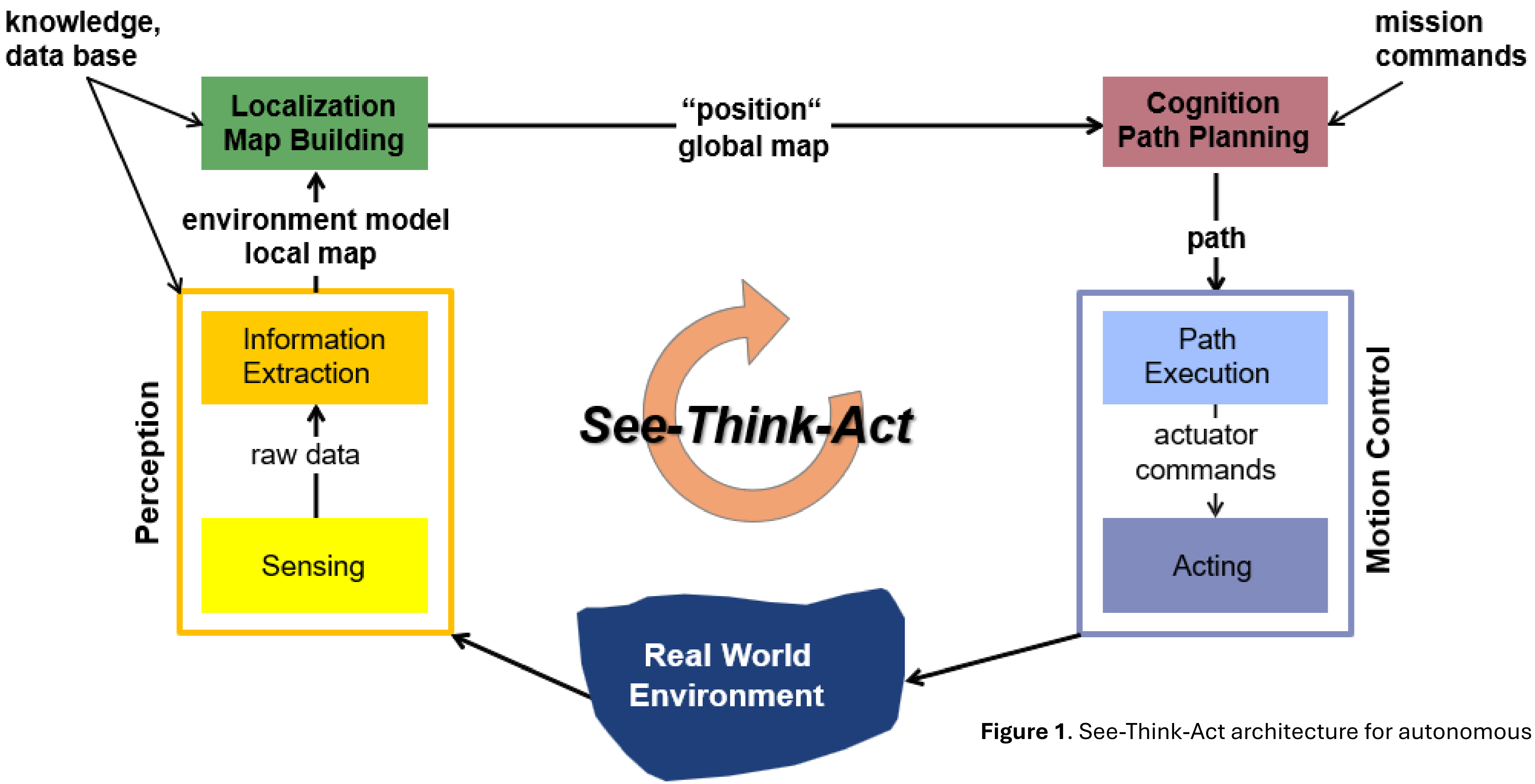


Figure 1. See-Think-Act architecture for autonomous vehicles [1].

Autonomous Vehicle System Architecture:

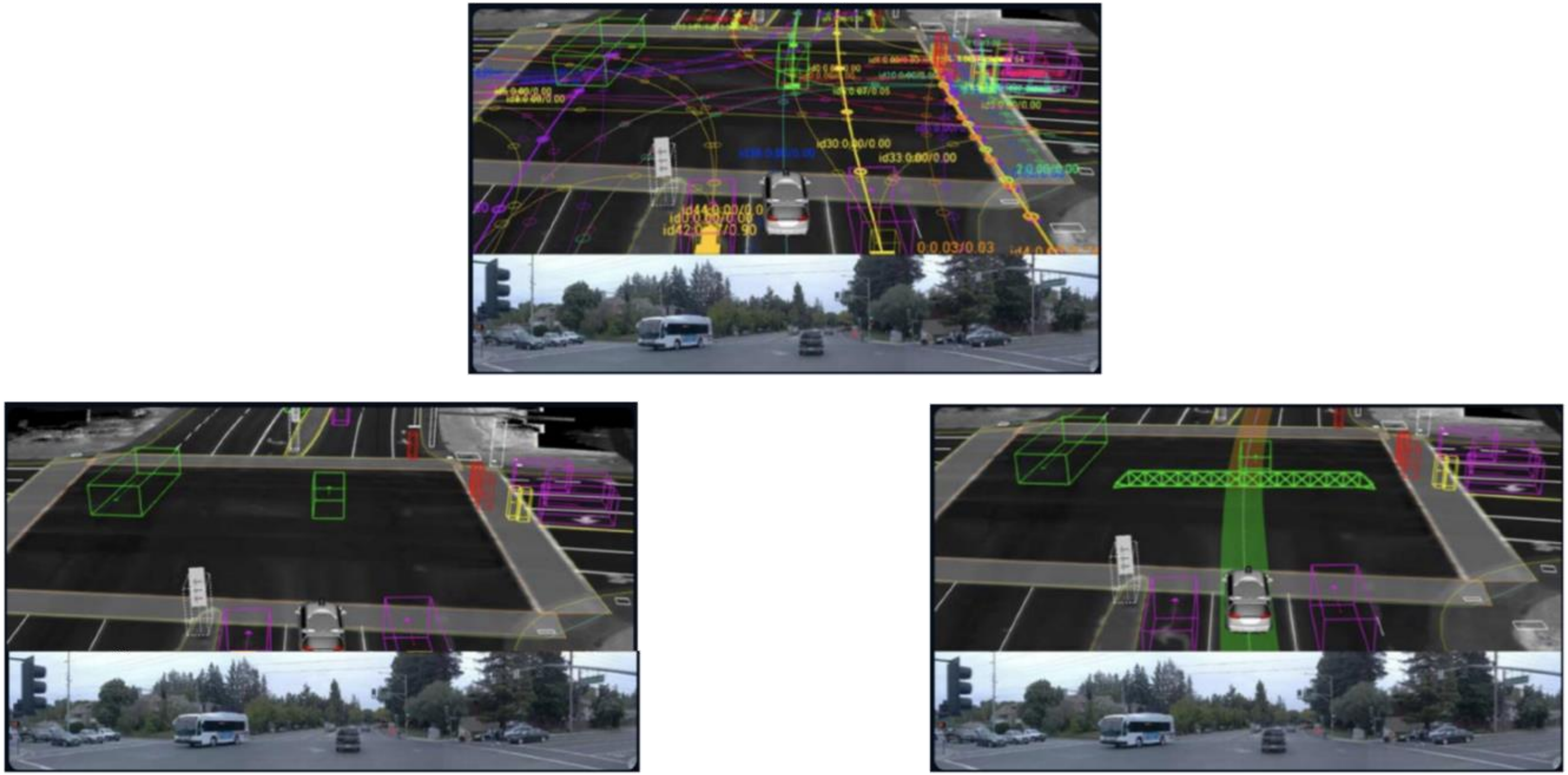


Figure 2. Autonomous vehicle perception, prediction, and trajectory planning at an urban intersection [2].

Hierarchy of Decision-Making Processes

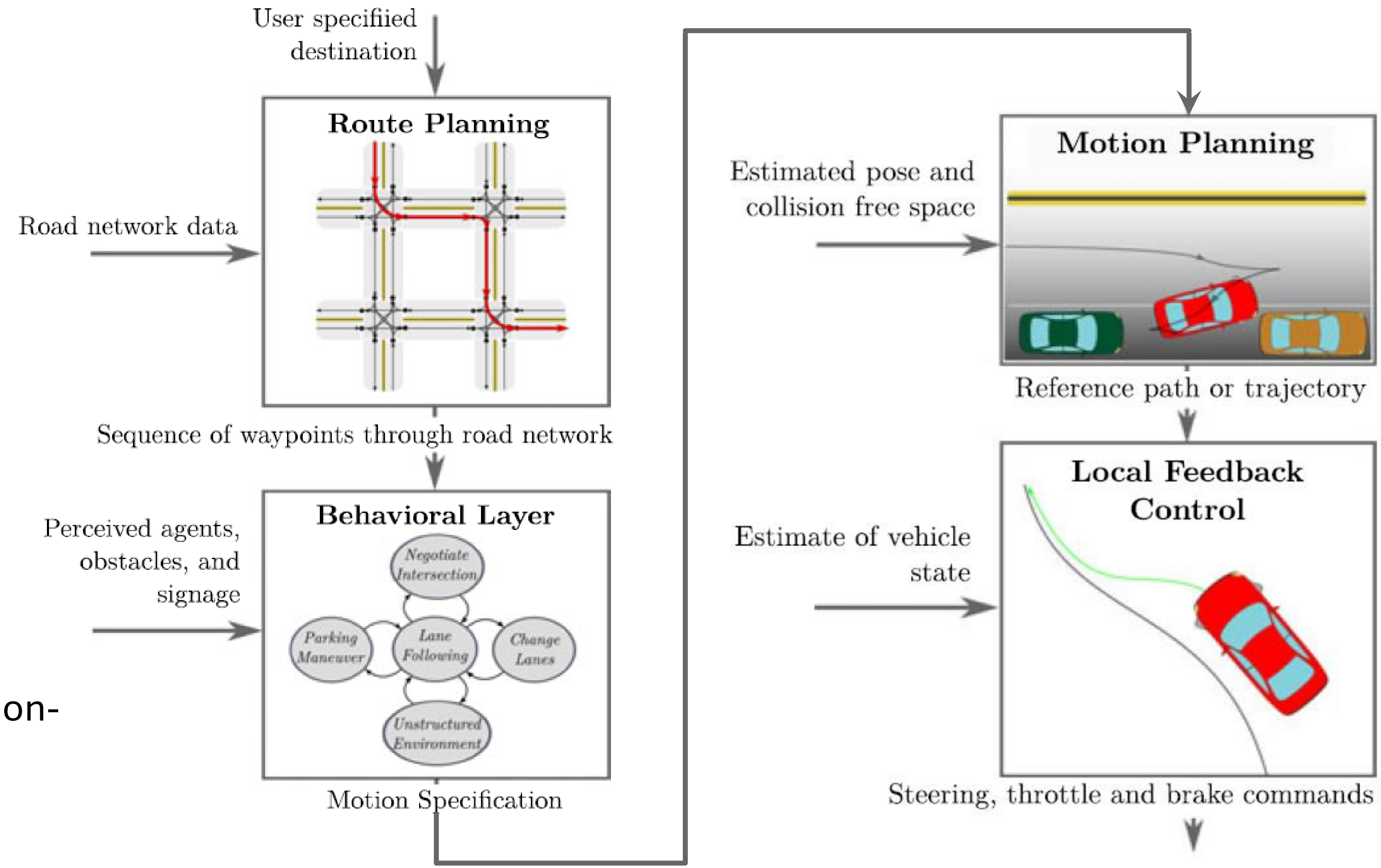


Figure. Hierarchy of decision-making processes [3]

What is a Control System?

- A system is an organized collection of parts that are highly integrated to accomplish an overall goal.
- The system has various ***inputs***, which go through certain processes to produce certain ***outputs***, which together, accomplish the overall desired goal for the system.
- **Control Technology**: The manipulation of an input to get the desired output.
- A ***control system*** manages and regulates the behavior of a system or process to achieve a desired outcome

Feedback Control System

- A **feedback control system** continuously compares the system output with a desired reference and adjusts its operation to reduce the difference.

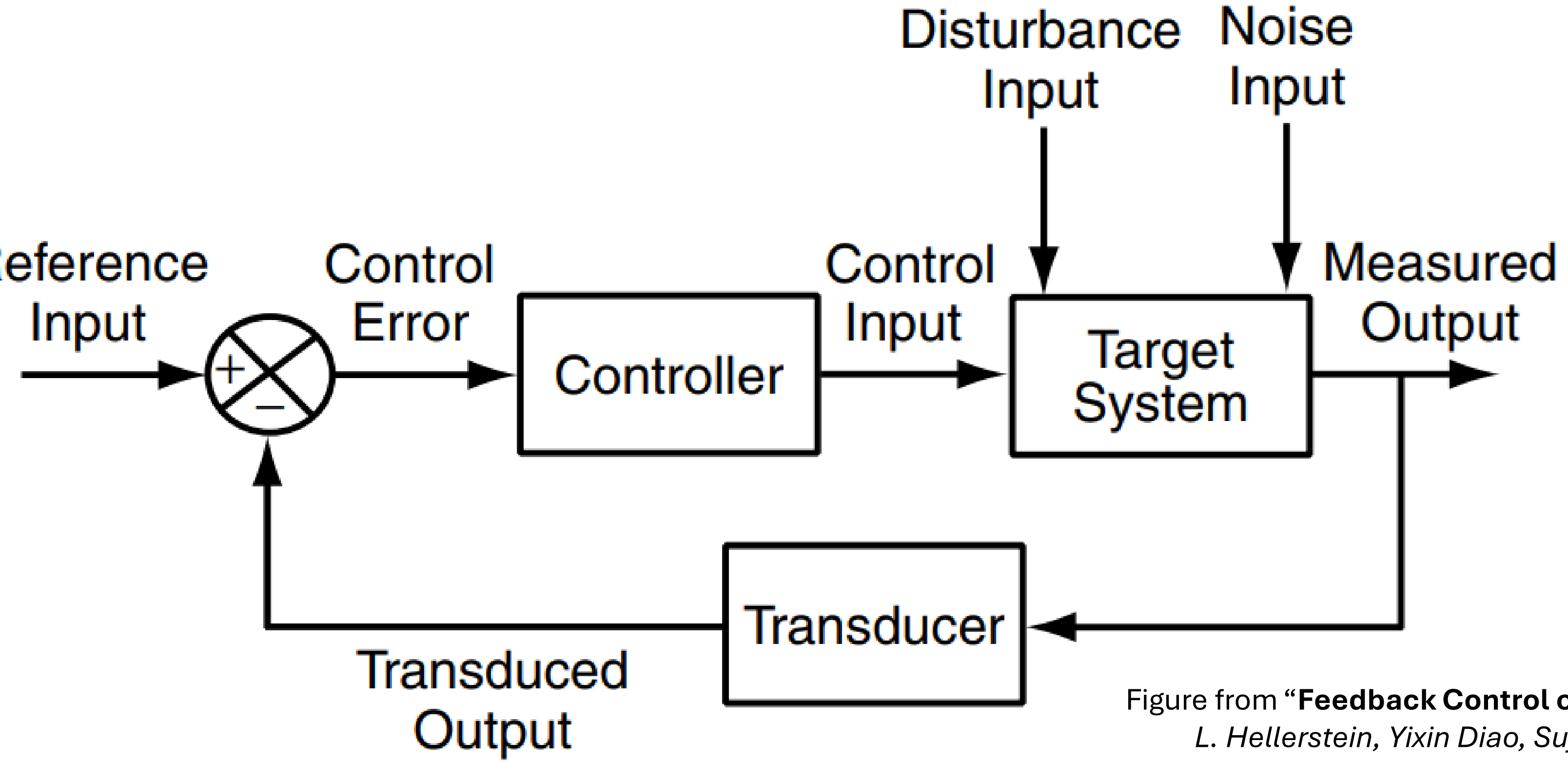


Figure from "**Feedback Control of Computing Systems**", *Joseph L. Hellerstein, Yixin Diao, Sujay Parekh, Dawn M. Tilbury*

Why Control Matters in AVs?

- Autonomous vehicles (AVs) must continuously regulate steering and speed to remain within lanes, obey speed limits, and maintain safety margins.
- Unlike human drivers, AVs rely on automatic feedback systems to respond to roadway geometry and disturbances.
- Control theory provides the mathematical and conceptual framework that ensures **safe, stable, and predictable vehicle motion**.
- From a civil engineering perspective, roadway design directly affects how difficult or easy this control problem becomes as sharp curves, poor sight distances, and inadequate superelevation increase control complexity.

Control Systems in Autonomous Vehicles

- **Autonomous vehicles integrate control systems** to execute planned motions and physically move the vehicle using actuators such as steering/throttle/braking.
- **Feedback control systems** measure vehicle states, compare actual behavior with desired behavior, and **continuously adjust steering, throttle, and braking** in real time.
- **Common feedback control methods:** Proportional–Integral–Derivative (PID), Linear Quadratic Regulator (LQR), Model Predictive Control (MPC)
- **AI and Machine Learning** support **high-level decision-making**, such as trajectory planning and obstacle avoidance in complex environments.

Civil Engineering Perspectives on Control Systems

- Road centerlines, lane markings, and horizontal/vertical alignments define the **reference trajectory** that the control system must follow.
- Design speed, curve radius, and superelevation impose **physical constraints** on safe vehicle motion based on friction and lateral acceleration limits.
- Control systems translate roadway geometric intent into precise vehicle actions (steering angle, acceleration).
- Poor geometric design (inconsistent curvature, inadequate transition curves, missing/faded markings) increases control complexity, tracking error, and reduces safety margins.
- **CE Design Impact:** A 200m radius curve requires $\sim 3\times$ more control effort than a 600m curve at the same speed (lateral acceleration $alat = \frac{v^2}{R}$).

Simulation & Validation

- **Autonomous vehicles are tested extensively in simulation** before deployment on real roads.
- **Digital Twin Simulation,** A virtual representation of a physical system designed to replicate real-world behavior and dynamics.
- **Simulation enables** testing of control and decision-making algorithms, performance evaluation under specific and extreme conditions, safe and repeatable experimentation
- **Real-Time Control Platforms** combine hardware and software to execute control algorithms **under strict real-time timing constraints**.
- **QUARC and QCar testbench** provide a platform for real-time control validation using both physical hardware and real-time computing systems.

Vehicle as a Dynamic System

- An AV is modeled as a moving physical system governed by Newton's laws of motion and tire-road interaction.

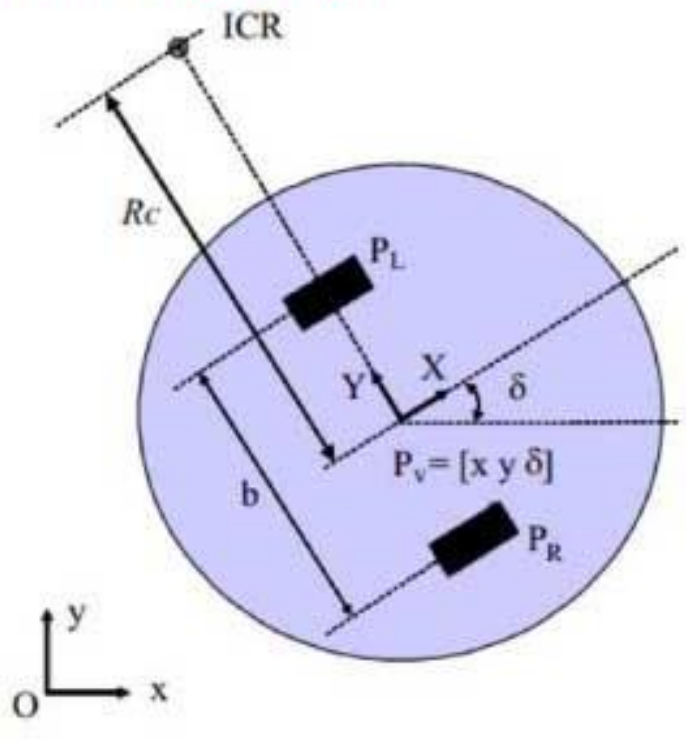
- **State variables** commonly include position (x, y), heading angle (ψ), longitudinal velocity (v), lateral velocity (ω) , and yaw rate.

- **Control inputs** include steering angle (δ) and longitudinal acceleration a

- Mathematical models (kinematic bicycle model, dynamic vehicle model) allow engineers to predict how the vehicle responds to control actions before deployment.

Kinematics of Differential Drive:

Reference point:
The middle of the two wheels



Differential kinematic configuration

Velocity of the reference point:

$$v = \frac{v_R + v_L}{2}$$

Angular velocity of the reference point:

$$\dot{\delta} = \frac{v_R - v_L}{b}$$

Kinematic configuration:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} v \cos\delta \\ v \sin\delta \\ \frac{v_R - v_L}{b} \end{bmatrix} = \begin{bmatrix} \frac{v_R + v_L}{2} \cos\delta \\ \frac{v_R + v_L}{2} \sin\delta \\ \frac{v_R - v_L}{b} \end{bmatrix}$$

https://matlab1.com/shop/matlab-code/mobile-robot-control-based-on-kinematics-of-differential-drive-dd/

Reference Path and Tracking Objective

- The roadway lane centerline (or planned trajectory) is treated as the **desired reference path** that the vehicle should follow.
- The AV's primary control objective is to minimize deviation from this path while maintaining stability and comfort.
- Deviations are quantified as **tracking errors,** measurable differences between desired and actual vehicle states.
- Continuous feedback correction of these errors ensures lane keeping and path following.
- **Path representation:** Often stored as sequence of waypoints (x_{ref}, y_{ref}) or parametric curves (Clothoids, splines).

Key Errors in Vehicle Control

- **Lateral error (e):** perpendicular distance between vehicle center of gravity and lane centerline (measured in meters).
- **Heading error (θ_e):** difference between vehicle orientation and road tangent direction (measured in radians).
- **Speed error (v_e):** deviation from desired or current speed limit.
- **Curvature error:** mismatch between vehicle path curvature and road curvature (advanced controllers).
- Control actions (steering, throttle, brake) are designed to drive these errors toward zero while respecting physical constraints.

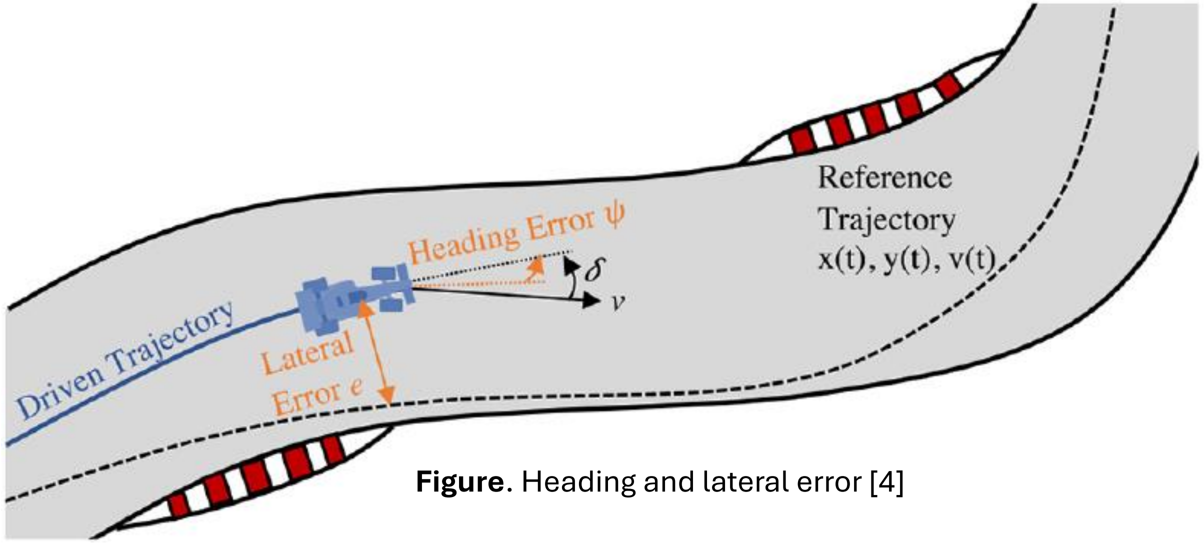
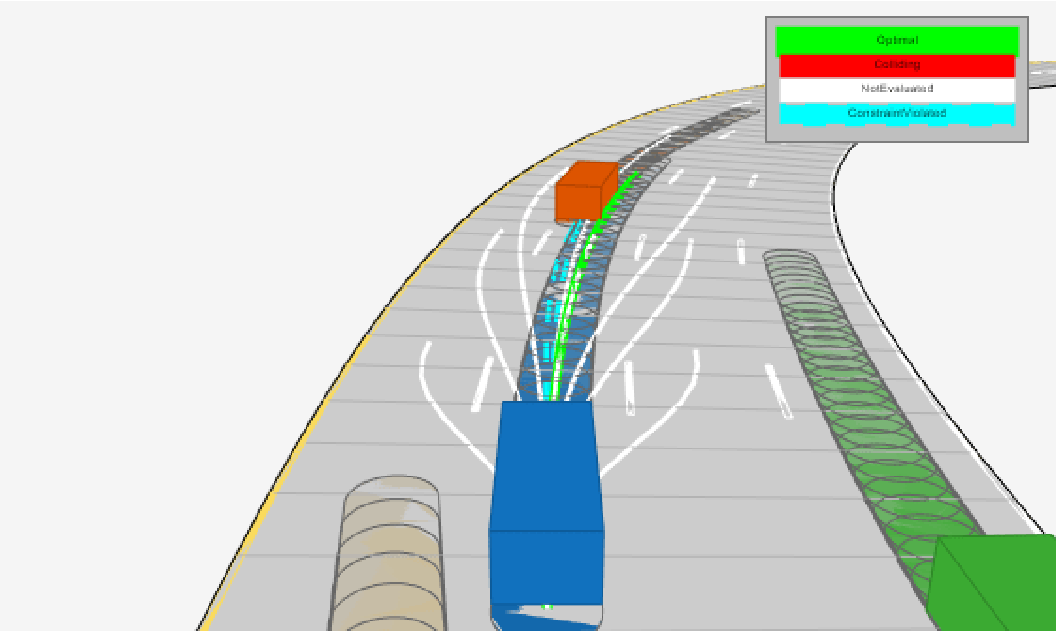


Figure. Heading and lateral error [4]

Control Theory Importance

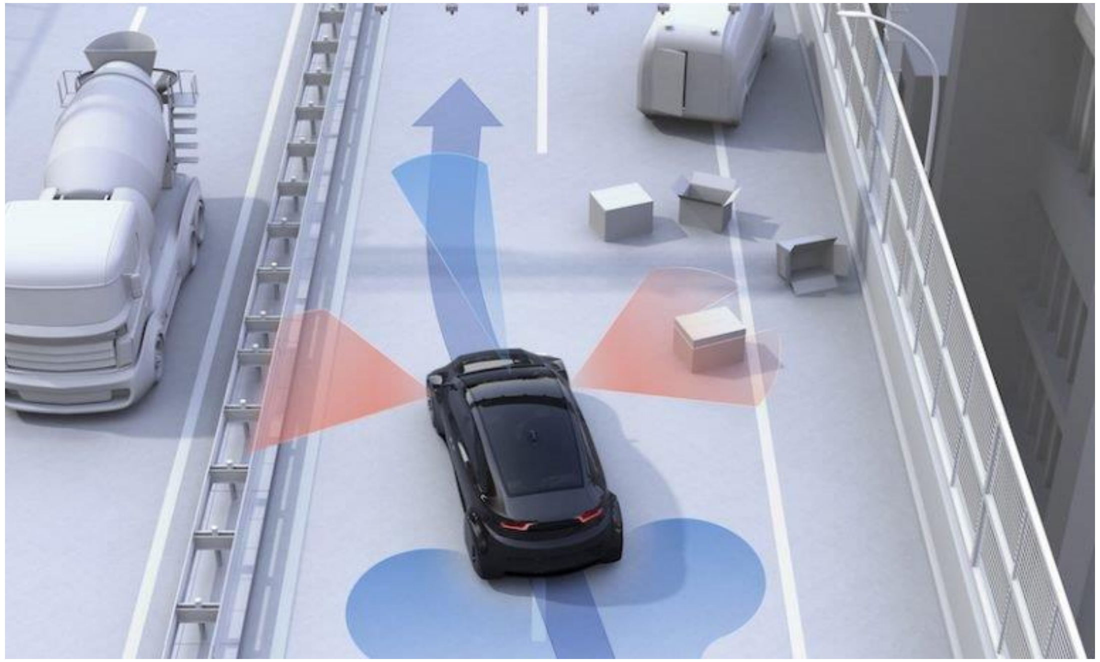
- **Reference trajectory:** represents the intended motion of the vehicle through the roadway. This reference becomes the **goal of the control system**
- **Roadway design** defines safe and intended vehicle motion with road lanes, and speed limits.
- Control theory determines:
  - How closely the vehicle follows the trajectory
  - How corrections are made when errors occur
- Control compares:
  - Desired motion (reference trajectory)
  - Actual motion (measured by sensors)
- Feedback is used to reduce error



Highway Trajectory Planning Using Frenet Reference Path - MATLAB & Simulink

Control Theory Importance

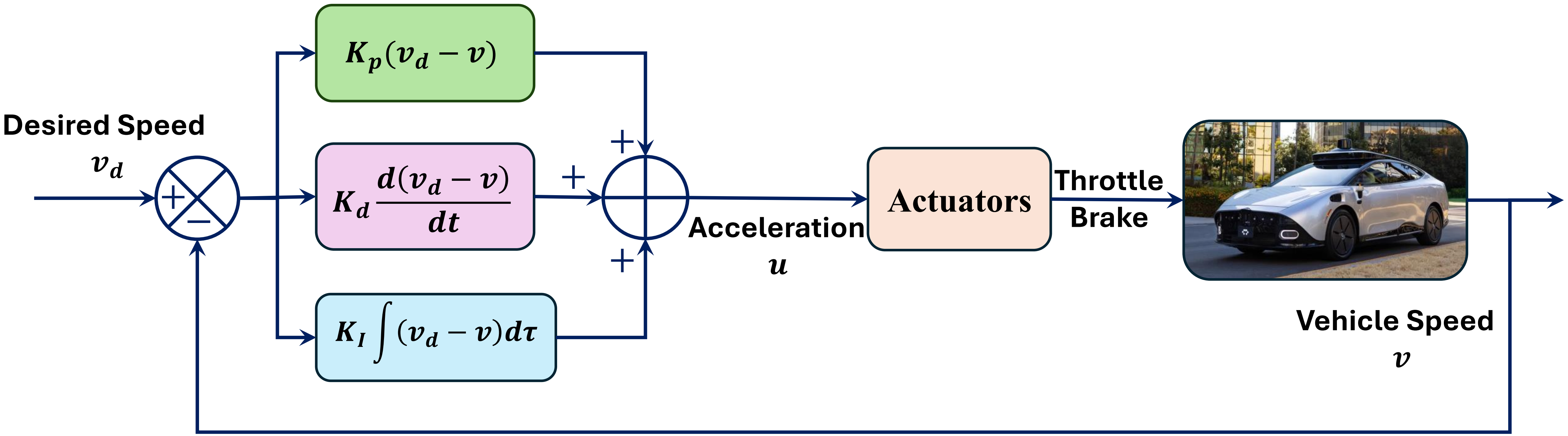
- Roadway safety in an AV relies on the control system
- Roadway safety **depends** on vehicles **staying within lanes** and **following traffic laws**
  - Safe speeds
  - Proper following distances
  - Road geometry defining expected movement
  - Curvature and grade
- These road features directly influence:
  - Steering effort
  - Speed limits
  - Vehicle stability



PID Control in AVs

- PID (Proportional–Integral–Derivative) control is a widely used feedback control systems that adjusts vehicle actions based on tracking error. It combines three actions:
- **Proportional (P)** responds to the **current error**, *If the vehicle drifts from the lane center, steer back proportionally*
- **Integral (I)** responds to **accumulated past error**, *corrects steady bias, such as road slope or misalignment*
- **Derivative (D)** responds to the **rate of change of error**, *dampens oscillations and smooths motion*
- **PID in Autonomous Vehicles,** lane keeping (steering control), speed and cruise control, adaptive braking
- **Why PID is Popular,** Simple and reliable, Easy to implement, Effective for many real-world vehicle control tasks

PID Control Example in Longitudinal Control



$$u = K_p(v_d - v) + K_I \int_0^t (v_d - v)d\tau + K_D \frac{d(v_d - v)}{dt}$$

Limitations of PID Control in AVs

- PID control does not anticipate upcoming road geometry or constraints.
- At high speeds (>25 m/s), delayed response can cause oscillations or overshoot, leading to discomfort or instability.
- Cannot explicitly handle hard constraints such as lane boundaries, actuator limits, or maximum lateral acceleration.
- Performance degrades significantly on sharp curves, during lane changes, or on low-friction surfaces (wet/icy roads).
- These limitations motivate predictive and optimization-based control approaches.

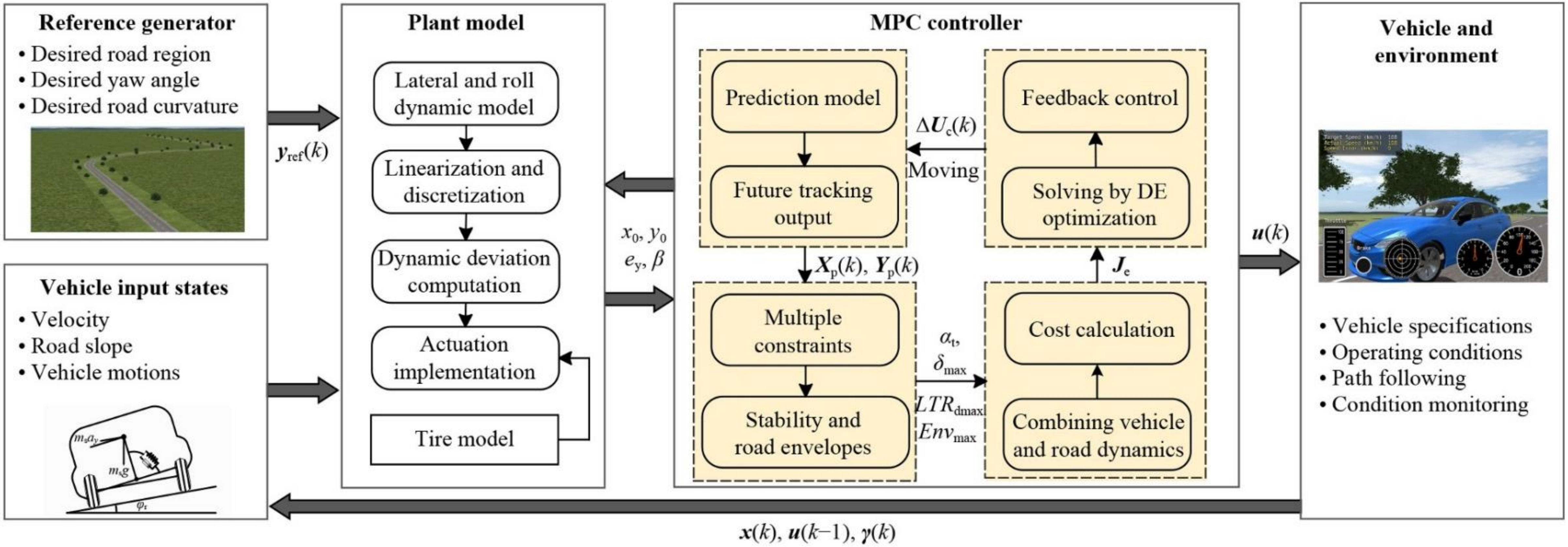
Model Predictive Control (MPC)

- MPC uses a vehicle dynamics model to **predict future motion** over a finite time horizon (e.g., 3 seconds = 30 time steps at 0.1s intervals).
- At each control cycle, MPC solves an **optimization problem**: find the best sequence of control inputs that minimizes tracking error while satisfying constraints.
- Only the **first control input** is applied; the process repeats continuously as new measurements arrive (receding horizon principle).
- MPC is **widely deployed** in commercial autonomous driving systems (Waymo, Tesla, Cruise) for trajectory tracking and collision avoidance.
- **Mathematical formulation:** Minimize $J = \Sigma(\text{tracking error}^2 + \text{control effort}^2)$ subject to dynamics and constraints.

MPC Importance in AVs

- **Explicit constraint handling:** Can directly encode lane boundaries, steering limits (±30°), acceleration limits (friction circle), and obstacle avoidance.
- **Smooth control outputs:** Optimization produces gradual steering and speed profiles, improving passenger comfort and reducing tire wear.
- **Multi-objective optimization:** Simultaneously balances safety (constraint satisfaction), efficiency (minimize energy), and comfort (minimize jerk).
- **Theoretical guarantees:** Under certain conditions, MPC provides provable stability and recursive feasibility.
- **Computational challenge:** Requires solving optimization problem in real-time (typically <100ms), demanding efficient algorithms and hardware.

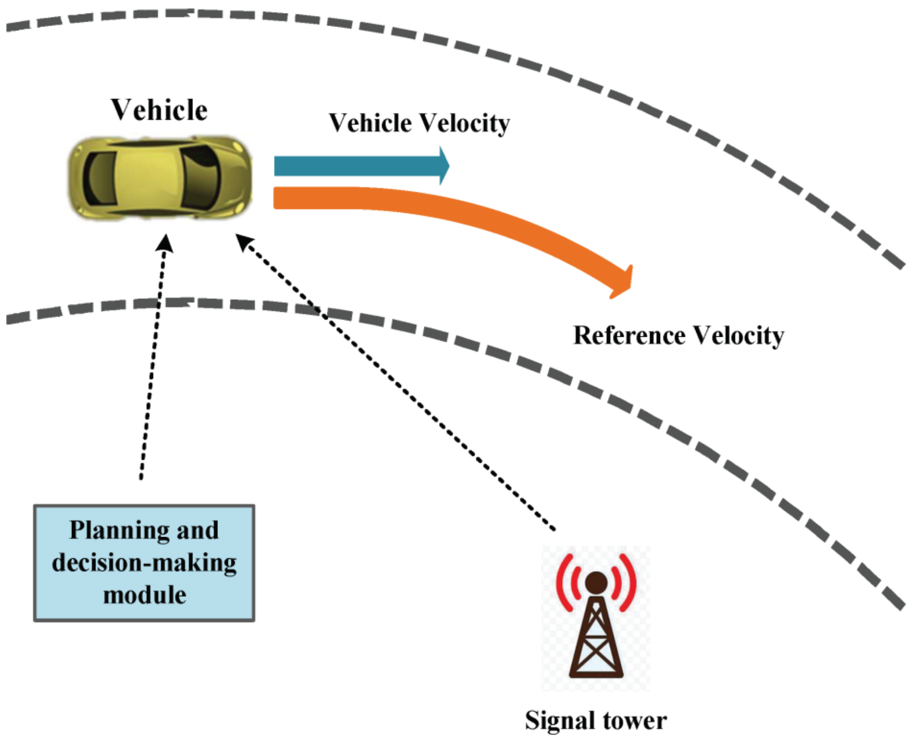
Example of Model Predictive Control



Extended MPC-based control scheme [5]

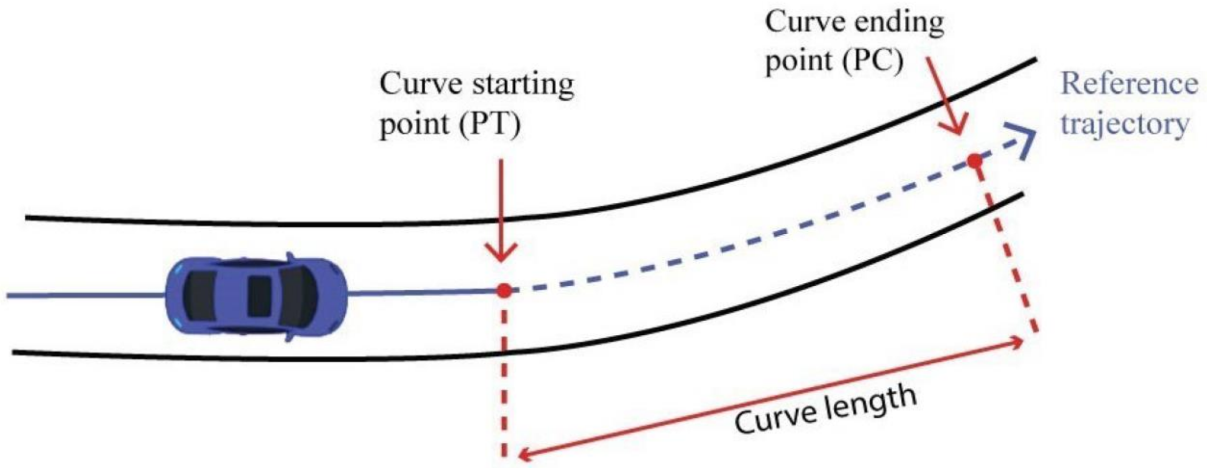
Example Control Techniques for AVs

Speed Control:



[Steering and Speed Control System Design for Autonomous Vehicles by Developing an Optimal Hybrid Controller to Track Reference Trajectory]

Steering Control:



[Efficient Nonlinear Model Predictive Control of Automated Vehicles]

Role of Civil Engineering in AV Control Systems

- **Infrastructure quality directly affects controllability:** High-quality pavement, clear markings, and consistent geometry reduce control difficulty.
- **Design decisions influence safety margins:** Generous curve radii, adequate sight distances, and proper superelevation provide wider operating envelopes.
- **Marking and signage:** Clear, well-maintained lane markings are critical for vision-based lateral control systems.
- **Connected infrastructure:** V2I (vehicle-to-infrastructure) communication can provide advance information (curve geometry, speed limits) to improve predictive control.
- **Civil engineering and control engineering are tightly coupled:** Optimal AV performance requires collaboration between disciplines.

References

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5. Liu, Q., Song, S., Hu, H. *et al.* Extended model predictive control scheme for smooth path following of autonomous vehicles. *Front. Mech. Eng.* **17**, 4 (2022). https://doi.org/10.1007/s11465-021-0660-4